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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2014

SECOND YEAR

Date : 27/05/2014 **Mathematics for Economics (General)**

Time : 11 am – 2 pm Paper : IV Full Marks : 75

(Use separate answer books for each group)

Group – A

1. Answer **any seven** questions of the following :- [7 × 5]

a) Consider the linear transformation $T: IP_2(\mathbb{R}) \to IP_2(\mathbb{R})$ defined by, $T(f(x)) = f'(x).(3+x) + 2f(x)$ compute the matrix representation $[T]_\beta$ where $\beta = \{1, x, x^2\}$.

b) Consider the linear transformation $T: IP_3(\mathbb{R}) \to M_{2\times2}(\mathbb{R})$ defined by, $T(f(x)) = \begin{pmatrix} f'(0) & 1 \\ 0 & 2f(1) \end{pmatrix}$. Is T invertible, give reasons.

c) Let V and W be vector spaces and let $T: V \to W$ be linear and if N(T) = { θ } where θ is the null vector of V then what type of mapping T is ?

d) Let V and W be vector spaces over a field F. Then prove that A linear mapping $T: V \to W$ is invertible if and only if T is one to one and onto.

e) Show that similar matrices have the same eigen value.

f) Prove that a real symmetric matrix is negative definite if and only if all its eigen values are negative.

g) Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to the normal form.

h) Let $T: P_2(\mathbb{R}) \to IP_2(\mathbb{R})$ be a linear operator defined by, $T(ax^2 + bx + c) = cx^2 + bx + a$. Then compute whether the matrix $[T]_\beta$ where $\beta = \{1, x, \frac{x^2}{2}\}$ is diagonalizable?

i) Let T is a linear operator on a vector space V. Show that the null space of T is an invariant subspace of V.

j) Let V be the vector space of all continuous real valued functions defined on [0, 1]. Then prove that the following is a inner product on V. $< f, g > = \int_0^1 f(t)g(t)\,dt$.

k) Let V be the vector space of all continuous real valued functions defined on [0, 1]. Consider the two inner products on V defined by, $< f, g >_1 \int_0^1 f(t)g(t)\,dt$ and $< f, g >_2 \int_{-1}^1 f(t)g(t)\,dt$

Prove that the vectors $f(x) = x^2$, $g(x) = x$ are orthogonal in the second inner product but not in the first.

2. Answer **any three** questions of the following :- [3 × 5]

a) Prove that the cube roots of unity form an abelian group w.r.t multiplication. [5]

b) Let $(G, \bullet)$ be a group and H, K are subgroups of $(G, \bullet)$ then $H \cap K$ is a subgroup of $(G, \bullet)$. [5]

c) Prove that, every subgroup of a cyclic group is cyclic. [5]

d) The set $\{a + b\sqrt{2} : a, b \in q\}$ forms a commutative ring with unity under addition and multiplication. Prove that this set forms a field. [5]

e) Define integral domain. Prove that a field is an integral domain. [1 + 4]

Group – B

3. Answer **any three** questions of the following :- [3 × 7]

a) Define a convex set. Prove that, the set of all feasible solutions of an L.P.P is a convex set [2 + 5]

b) Solve the following L.P.P by graphical method [7]

minimize Z = 3x + 4y

Subject to
$$5x + 4y \geq 20$$
$$-x + y \leq 3$$
$$x \leq 4$$
$$y \geq 3 \quad \text{where}\,(x, y) \geq 0$$

c) Use charne's Big M method to solve the following L.P.P. [7]

Maximize $Z = x_1 + 5x_2$

Subject to
$$3x_1 + 4x_2 \leq 6$$
$$x_1 + 3x_2 \geq 3$$
and $x_1, x_2 \geq 0$

d) Use duality and simplex method to find the optimal solution of the following L.P.P. [7]

Maximize $Z = 3x_1 + 4x_2$

Subject to
$$x_1 + x_2 \leq 10$$
$$2x_1 + 3x_2 \leq 18$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$
and $x_1, x_2 \geq 0$

e) State the fundamental theorem of Duality. Prove that the dual of the dual is the primal. [2 + 5]

4. Answer **any one** question of the following :- [1 × 4]

a) Show that the feasible solution (1, 0, 1, 6)) of the system
$$x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 - x_4 = 0$$
is not basic.

b) Food X contains 5 units of vitamin A and 6 units of vitamin B per gram and costs 20 p/gm. Food Y contains 8 units of vitamin A and 10 units of vitamin B per gram and costs 30 p/gm. The daily requirements of A and B are at least 80 and 100 units respectively. Formulate the above as a linear programming problem to minimize the cost.